Department of Mechanical and Industrial Engineering

Northeastern University

Handbook for the Preliminary Examination of the Doctoral Qualifying Exam

August 3, 2012

The following pages contain specific information and sample problems which can be useful when you prepare for the Preliminary Exam, which is the first step of the Doctoral Qualifying Examinations. The only revision from the 2009 edition is that the Fundamentals of Polymers section (D5) has been added.

See http://www.coe.neu.edu/gse/sc_mime_phd.html for general information.
Section A: Applied Mathematics

A1 Engineering Mathematics

Ordinary Differential Equations using exact methods, series and transforms;

Partial Differential Equations using separation of variables (Fourier series, eigenfunction expansions) and transforms;

Linear Algebra Matrices and linear equations, determinants, eigenvalue problems;

Vector Field Theory including Cartesian, cylindrical and spherical coordinates, gradient, divergence and curl, and Integral Theorems (Divergence Theorem, Stokes’ Theorem).

Introductory Complex Variables Including complex algebra (Cartesian and polar form of complex numbers) roots, powers, elementary transcendental functions and their inverses (complex exponential, complex logarithm, trigonometric and hyperbolic functions). Multi-valued functions, branches and branch cuts.

References for preparation:


A2 Engineering Computation

Numerical solution of non-linear algebraic equations (root-finding techniques),
Matrix analysis, curve fitting,
Numerical integration and differentiation,
Numerical solutions of ordinary differential equations,
Finite difference techniques as applied to elliptic,
Parabolic and hyperbolic partial differential equations,
The finite element method,
The Galerkin method

References: B. Camahan, H.A. Luther and J. Wilkes "Applied Numerical Methods",
Introduction to Finite Element Techniques", P. Tong and J.N. Rossettos (1977)
"Finite Element Method - Basic Technique & Implementation" The MIT Press,
Cambridge MA.
A3 Probability and Statistics
Discrete and continuous random variables.
Cumulative probability distributions and moment generating functions.
Expectation of random variables.
Discrete and continuous probability distributions including: binomial, Poisson, geometric, uniform, exponential and normal.
Multivariate probability distributions, covariance and independence of random variables.
Sampling distributions and limiting theorems.
Parameter estimation.
Confidence intervals and hypothesis testing.
Regression and ANOVA.
Chi-squared and non-parametric tests.

ODEs

1. Find the first six terms of the power series solution of the following equation:

\[ y'' - xy = 0. \]

State the interval of convergence of the full infinite series. Compute the Wronskian at \( x = 0 \) to show that two linearly independent solutions are obtained.

2. Find the general solution of

\[ x^2 y'' - x(1 + x) y' + y = 0. \]

Vectors

1. Stokes’ Flow in the vicinity of a sphere of unit radius is given by

\[ V = \left( 1 - \frac{3}{4} \rho \right) \frac{1}{\rho^2} \mathbf{U} - \frac{3}{4} \rho (1 - \frac{1}{\rho^2}) \left( \mathbf{U} \cdot \hat{e}_\rho \right) \hat{e}_\rho \]

where \( \hat{e}_\rho \) is the radial unit vector, \( \rho \) is the radial spherical coordinate and \( \mathbf{U} \) is the fluid velocity far from the sphere.

Consider a spherical coordinate system so that \( \mathbf{U} \) is directed along the positive \( z \)-axis. Further consider the closed surface \( S \) composed of three parts:

- \( S_1 \) is a hemispherical surface defined by \( \rho = R > 1, 0 \leq \phi \leq \pi/2, 0 \leq \theta \leq 2\pi \);
- \( S_2 \) is the surface of the sphere itself, defined by \( \rho = 1, 0 \leq \phi \leq \pi/2, 0 \leq \theta \leq 2\pi \) and
- \( S_3 \) the annular region in the \( x - y \) plane given by \( 1 \leq \rho \leq R, \phi = \pi/2, 0 \leq \theta \leq 2\pi \).

Evaluate the surface integral \( I = \int_S \mathbf{V} \cdot \hat{n} dA \), on \( S_3 \) where \( \hat{n} \) is the outward pointing normal.

Evaluate \( I \) for the surface \( S_2 \).

Verify the divergence theorem

\[ \int_S \mathbf{V} \cdot \hat{n} dA = \int_{\mathcal{V}} \nabla \cdot \mathbf{V} d\mathcal{V}, \]

where \( \mathcal{V} \) is the volume enclosed by \( S \).
2. Show that the line integral

\[ I = \int F \cdot dr = \int ydx + (x + z e^{yz})dy + ye^{yz}dz \]

is path independent. Find the scalar field, \( \phi(x, y, z) \), from which \( F \) is derived. Evaluate \( I \) between \((1,2,3)\) and \((-10,5,-4)\).

3. Let \( S \) be the portion of the unit sphere \( x^2 + y^2 + z^2 = 1 \) lying above the \( x - y \) plane and bonded by the unit circle \( x^2 + y^2 = 1 \) in the \( x - y \) plane. Verify Stokes’ Theorem,

\[ \int_S \nabla \times F \cdot \hat{n}dA = \int_C F \cdot dR, \]

for the vector field \( F = \nabla \times (z^2 \hat{e}_r) \) by performing the surface integral and line integral separately. Draw a clear sketch of the surface showing a) the outward pointing normal, (away from the origin), b) the bounding curve \( C \) and c) the positive sense in which the line integral is to be performed.

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### Laplace Transform

1. Using the Laplace transform, solve the initial value problem.

\[ \ddot{x} + \omega^2 x = H(1 - t)e^{1-t}; \quad \dot{x}(0) = x(0) = 0 \]

for \( x(t) \), where \( H(t) \) is the Heaviside function.

2. Using the Laplace transform solve,

\[ \ddot{x} - x = 1 + e^{3t}; \quad x(0) = x_0; \quad \dot{x}(0) = v_0. \]

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### Complex Variables

1. Find and plot all roots of the following equations:

a. \( z^4 - 6z^2 + 25 = 0 \)  
b. \( \cos z - 2i = 0. \)

2. Let \( f(z) = z^i \) and \( g(z) = i^z. \) Write these two functions in terms of the complex logarithm and the complex exponential. What is \( f(i) - g(i) \)?

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### Partial Differential Equations

1. Transform the partial differential equation,

\[ \frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial x^2} \]
with boundary and initial conditions given by

\[ \theta(x, 0) = \lim_{x \to \infty} \theta(x, t) = 0; \quad \theta(0, t) = 1, \]

into an ordinary differential equation using the similarity variable \( \eta = x/\sqrt{\alpha t} \), and assuming that \( \theta(x, t) \equiv f[\eta(x, t)] \).

Find the boundary conditions that \( f(\eta) \) must satisfy and solve the resulting boundary value problem.

2. Using separation of variables, solve

\[ \frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial x^2}, \quad 0 \leq t < \infty, \quad -1/2 \leq x \leq 1/2 \]

for \( \theta(x, t) \), with the initial condition \( \theta(x, 0) = 1 \) and the boundary conditions \( \theta(\pm 1/2, t) = 0 \).

**Linear Algebra**

1. Consider the matrix equation \( A \mathbf{x} = \mathbf{b} \), where \( \mathbf{x} = (x_1, x_2)^T \), \( \mathbf{b} = (1, 10)^T \) and

\[ A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \]

a. Show that a unique solution exists.
b. Find the eigenvalues and eigenvectors of the matrix \( A \) and show that the eigenvectors are orthogonal.
c. Solve for \( \mathbf{x} \) using an eigenvector expansion.

2. Consider the linear system of equations:

\[
\begin{align*}
 x_1 + 2x_2 + 3x_3 + 3x_4 &= 1 \\
 2x_1 + 5x_2 + 10x_3 + 7x_4 &= 4 \\
 2x_1 + 5x_2 + 9x_3 + 5x_4 &= 1 \\
 x_1 + 2x_2 + 3x_3 + 4x_4 &= 2
\end{align*}
\]

a. What condition on the matrix of coefficients determines whether a unique solution exists?
b. Solve for the values \( x_1, x_2, x_3 \) and \( x_4 \) using Gaussian Elimination.
1. (25 minutes)

Use the Newton-Raphson method to calculate one of the eigenvalues of the matrix

\[
\begin{pmatrix}
1 & 2 & -1 \\
0 & 2 & 1 \\
1 & -1 & 0
\end{pmatrix}
\]

Take \( x = 2 \) as your initial guess and do 4 iterations.

*Newton-Raphson Formula*

\[
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}
\]

2. (25 minutes)

Evaluate the following integral using Romberg method.

\[\int_0^\pi \cos x \, dx\]  (RADIANS)

Start with the trapezoidal rule with one, two and four intervals.

*Romberg Method*

Improved Value = More Accurate + \( \frac{1}{4^l - 1} \) (More Accurate - Less Accurate)

or

\[I_l^n = I_{l,1}^n + \frac{1}{4^l - 1} [I_{l,1}^n - I_{l-1}^n]\]
A3 Engineering Probability and Statistics

1. (30 minutes)

The diameter of holes for a cable harness is known to have a standard deviation of 0.01 in. A random sample of size 10 yields an average diameter of 1.5045 in. Use $\alpha = 0.01$.

a. Test the hypothesis that the true mean whole diameter equals 1.5 in.
b. What sample size would be necessary to detect a true mean hole diameter of 1.505 in. with a probability of at least 0.90?
c. What is the type II error if the true mean hole diameter is 11.505 in.?

2. (30 minutes)

In a laboratory experimental study, it is desirable to run careful checks on the variability of readings produced on standard samples. In a study of the amount of calcium in drinking water undertaken as part of a water-quality assessment, the same standard was run through the laboratory six times at random intervals. The six readings, in parts per million from an area (A), were 9.45, 9.61, 9.32, 9.48, 9.70, and 9.26.

a. Estimate the population variance for readings on this standard, using a 90% confidence interval.
b. A second sample of size 9 from another area (B) gave sample variance $S^2=0.0231$. Estimate the true (population) variance ratio of area A over Area B with 90% confidence.
c. Justify the validity of pdf you used in parts a and b.
Group B

B1 Thermodynamics

Topics:
- First and Second Law of Thermodynamics
- Energy, Available Energy and Entropy
- Temperature and Pressure
- Work and Heat Interaction
- Heat Engine
- Characteristic Function
- Simple System
- Thermophysical Properties of Pure Substances
- Equation of State
- Conversion Devices
- Power Generation
- Refrigeration and Energy Pump
- Chemical Reaction and Chemical Equilibrium


B2 Fluid Mechanics

Finite Control Volume Analyses
- Reynolds Transport Theorem; Continuity;
- Conservation of momentum; Conservation of energy;
- Bernoulli equation

Differential Control Volume Analyses
- Navier-Stokes equations; Laminar flow analyses;
- Boundary layer analyses; Potential flow analyses

Similitude
- PI theorem; Non-dimensional parameters;
- Model flow vs prototype flow;
- Non-dimensionalization of governing equations and boundary conditions

B3 Heat Transfer

Conduction
Steady State in multi-dimensional system (development and solution)
Problems involving internal heat generation source
Extended surface problems, including fin efficiency
Extended surface problems with varying cross section area, (Bessel function solutions)
Transient problems, lumped capacitance and multi-dimensional systems

Convection
Definition of the heat transfer coefficient
Use of correlations
Hydrodynamic and thermal boundary layer
Use of the integral analysis to calculate the heat transfer coefficient: natural convection
Application of the Navier-Stokes equations to the convection problem

Radiation
Use of the Stefan-Boltzmann Law, Planck's distribution law and black body emissivity functions
Emissivity, reflectivity and transmittance definitions and their use in spectral and gray surfaces
Kirchoff's Law and multi-band width problems
Definition and use of view factors
Solution of multi-surface problems
Combined modes of heat transfer problems

References: F.P. Incropera & D.P. DeWitt "Introduction to Heat Transfer", Wiley,
Corporation, S. Kakac & Y. Yener "Convective Heat Transfer", Hemisphere
B1 Thermodynamics

1. **(30 minutes)**

A new vehicle propulsion scheme calls for the use of liquid nitrogen as "fuel". The details of the power cycle are not the issue here (basically, the liquid nitrogen is heated in contact with the atmosphere, pressurizes, and expanded through the turbine that drives the vehicle). You are asked to calculate the maximum work that could theoretically be derived from the liquid nitrogen fuel. During each refueling stop, the driver purchases a dewar vessel (a bottle) containing 0.5 m$^3$ of liquid nitrogen at atmospheric pressure. He leaves in exchange a used bottle, that is, a bottle containing gaseous N$_2$ at atmospheric pressure and temperature. The properties of nitrogen as saturated liquid at 1 atm are:

\[ v = (1.24)10^{-3} \text{ m}^3/\text{kg}, \ h = -121.5 \text{ kJ/kg}, \ s = 2.85 \text{ kJ/kg K} \]

The corresponding properties of nitrogen at atmospheric temperature and pressure (298K, 1 atm) are

\[ v = 0.49 \text{ m}^3/\text{kg}, \ h = 172.1 \text{ kJ/kg}, \ s = 6.25 \text{ kJ/kg K} \]

2. **(25 minutes)**

A 12 volt battery of an urban bus is able to deliver to the engine starter an average current of 600 Amperes for 1 minute. The resulting available electrical work is sufficient to start the engine. Now suppose that instead of using the battery an attempt will be made to start the engine with compressed air from the air tanks of the bus. If the pressure of the air in the tanks is 10 atm and the temperature 25°C estimate the volume of the air tanks that will be needed to start the engine.
B2: Fluid Mechanics

Problem 1. [60]

For a steady, incompressible ($\rho = $ density), two dimensional flow (in xy plane) with negligible body force, the velocity vector, $\vec{V}$, is $\vec{V} = u \cdot \hat{i} + v \cdot \hat{j}$, $u = 2y$, $v = -2x$.

(1.) Find the stream function, $\Psi$, and sketch a streamline passing through a point (1, 1).

(2.) Show that the flow satisfies the continuity.

(3.) Is the flow irrotational?

(4.) Find the pressure gradient, $\nabla p$.

(5.) Show that $p$ is constant along a streamline.

(6.) Find $V_r$ and $V_\theta$ in the cylindrical coordinates $(r, \theta)$.

Problem 2. [40]

Consider a steady, incompressible ($\rho = $ density), laminar ($\mu = $ dynamic viscosity), fully-developed, two-dimensional (velocity vector, $\vec{V} = u \cdot \hat{i} + v \cdot \hat{j}$) flow between two horizontal plates (top plate at $y = H$ moving at the velocity $U \cdot \hat{j}$, bottom plate at $y = 0$ being stationary) with gravity ($g$) acting in $-y$ direction. The flow is driven by a specified pressure gradient, $\partial p/\partial x$, as well as by the motion of the top plate.

(1.) Show $u = u(y)$, $v = 0$.

(2.) Show that the pressure variation in $y$-direction is hydrostatic; i.e. $p = p_{HS} + p_{HD}$ with $\partial p_{HD}/\partial y = 0$ ($p_{HS} = $ hydrostatic pressure component, $p_{HD} = $ hydrodynamic pressure component).

(3.) Write a governing equation and boundary conditions for the velocity field.

(4.) Find a non-dimensional velocity profile, $\bar{u} = \bar{u}(\bar{y})$ with $\bar{u} = u/U$, $\bar{y} = y/H$. 
B3: Heat Transfer

PROBLEM 1: Consider a long thin-walled cylinder of radius $r_0$. The cylinder rotates steadily with an angular velocity $\omega$ as illustrated in the figure below. One half of the cylinder passes through a furnace where it is heated by exposing it to a hot gas at temperature $T_g$, while the other half is cooled by convection in open air at temperature $T_\infty$, with constant heat transfer coefficients $h_1$ and $h_2$, respectively. Develop a formulation of the problem (i.e., obtain differential equation(s) and state boundary conditions) for the determination of the steady temperature $T(s)$ in the cylinder.

PROBLEM 2: Consider an isothermal viscous fluid of temperature $T_0$ between two large parallel plates separated by a distance $L$ as illustrated in the figure below. The upper plate is moving with a constant velocity $U$ in the $x$-direction and is thermally insulated. The lower plate is stationary and a step change in its temperature from $T_0$ to $T_w$ occurs at $x = 0$.

(a) Show that the velocity distribution in the fluid is given by

$$U(y) = \frac{U}{L} y$$

(b) Assuming a linear temperature distribution in the thermal boundary layer, obtain an expression, by integral method, for the local heat transfer coefficient $h_x$ in the region where $\delta_T < L$. 
**Problem 3:** A furnace, in the form of a cylindrical cavity of depth $L = 15$ cm and diameter $D = 10$ cm, is open at one end to surroundings at $27^\circ$C. The side and bottom surfaces are heated electrically and well insulated. Assuming that the inside surfaces are gray with an emissivity $\varepsilon = 0.7$, calculate the average heater power required to maintain the inside surfaces at a mean temperature of $1527^\circ$C.
Section C: Applied Mechanics

C1: Dynamics and Vibrations
Basic concepts of rigid body kinematics and kinetics.
Newton's Laws of Motion.
Energy and momentum methods for particles and rigid bodies.
Free and forced vibrations of single and multiple degree of freedom systems
Eigenvalue problems and modal expansions in vibration
Simple vibration of rods (longitudinal and torsional) and beams (bending).

References:

C2: Mechanics of Deformable Bodies
- Basic concepts of stress and strain and stress-strain relations.
- Yield strength and elastic-perfectly-plastic material behavior.
- Transformation of stress, principal stresses in three dimensions, Mohr’s circles.
- Boundary and continuity conditions for three-dimensional continua.
- Structural mechanics of bars, shafts, and beams under axial, torsional, and transverse loadings.
- Energy methods (including Castigliano’s Theorem) and calculation of deflections (including shear deformation) of beams, frames, and rings for statically determinate and statically indeterminate loadings.
- Thin-walled pressure vessels.
- Stability of structures; buckling of columns and structures.

References:

C3 Dynamic Systems and Control
- First-order and second-order system concepts such as: time constants; natural frequency; damping ratio; damped frequency; impulse, step and ramp response; and steady state error.
- Lumped models of electromechanical components, such as: DC motors; RLC circuits; gears; torsional and translational inertias; ball screws; and rack and pinion.

You should be able to synthesize a set of first-order ordinary differential equations (ODEs) that model the dynamic behavior of a collection of the above components. Furthermore, second-order
systems and extensions to higher order systems should be mastered in the context of the following:

- Laplace transforms
- Superposition and how it applies to linear dynamic systems
- Time-domain simulation techniques for dynamic systems
- The purpose of feedback, basic properties of feedback and principles of PID controllers.
- Dynamic models and ODEs in state space representation
- Dynamic response, transient response characteristics and dominant behavior
- Stability and Routh-Hurwitz criterion
- Root locus analysis and design
- Frequency response design
- Nyquist stability criterion
- Fundamentals of lead, lag and lead-lag design using root locus and frequency response techniques

References:

C4: Finite Element Method

- Weighted residual methods and identification of essential and natural boundary conditions.
- Implementation of variational methods, such as Rayleigh-Ritz, Galerkin, Petrov-Galerkin to 1D boundary value problems.
- Derivation of interpolation functions and $C^n$ continuity.
- Truss, beam, plate shell, and solid elements and element defects.
- Global stiffness matrix, assembly of element equations, numerical implementation of boundary conditions.
- Isoparametric elements and numerical integration.
- Solution of transient problems with implicit and explicit methods.
- Application of finite element method in fluid heat transfer problems.

References:
Sample Problems

C1: Dynamics and Vibrations

Problem C1-1: Consider a rigid bar with a length \( l \) and mass of \( m \) supported by a pin on one end and a linear spring at the other end. The system is initially at rest with the bar at horizontal position. A small particle with mass \( m_0 \) then is dropped from a distance \( h \) from the bar and hit the bar and stuck to it. Assuming small displacement for the bar, derive the equation of motion of the bar around its equilibrium position. Find the displacement of the bar as a function of time. Note the gravity acts on the system. Simplify your answer when \( h \rightarrow 0 \).

Problem C1-2: A robot arm is manipulating a delicate workpiece in the form of a thin disk with mass \( m \) and radius \( r \), as sketched. At the instant shown, all members are in the plane of the sketch, i.e. the XY plane. The jaws is rotating respect to the arm with angular velocity and acceleration of \( \omega_1 \) and \( \dot{\omega}_1 \). The arm is rotating with respect to the body of the robot at angular velocity and angular acceleration of \( \omega_2 \) and \( \dot{\omega}_2 \). The whole mechanism is rotating with respect to the ground with \( \omega_3 \) and \( \dot{\omega}_3 \). At the instant shown, \( \theta = 30^\circ \) and \( \phi = 45^\circ \). Find the forces exerted to the workpiece at that instant.
**Problem C2-1**: The uniform bar with Young’s modulus \( E \) and Poisson’s ratio \( \nu \) of length \( L \) and square cross-section \((a \times a)\) fits perfectly into the frictionless and rigid channel. If the compressive normal stress \( p \) is applied as shown, determine the
a) the change in length of the bar, and
b) the change in the cross-section dimensions.
c) If the yield strength is \( Y \), determine the value of \( p \) which causes yielding using the Tresca yield criterion.

**Problem C2-2**: The semicircular ring (BC) of radius \( R \) and flexural rigidity \( EI \) is fixed at end C. The other end is attached to a rigid member (AB). If the force \( P \) is applied at an angle \( \alpha \) as shown, determine the vertical deflection of pt. A. You may find the integration table below useful.

\[
\int \sin \phi \cos \phi d\phi = -\frac{1}{4} \cos 2\phi, \quad \int \cos^2 \phi d\phi = \frac{1}{4} (2\phi + \sin 2\phi), \quad \int \sin^2 \phi d\phi = \frac{1}{4} (2\phi - \sin 2\phi),
\]
\[
\int (1 - \cos \phi)^2 d\phi = \frac{3\phi}{2} - 2\sin \phi + \frac{1}{4} \sin 2\phi, \quad \int (1 - \sin \phi)^2 d\phi = \frac{3\phi}{2} + 2\cos \phi - \frac{1}{4} \sin 2\phi
\]

**Problem C2-3**: The two bars are attached as shown as subjected to the force \( P \) at the midpoint. The Young’s moduli, cross-sectional areas, and yield strengths are shown.
a) Determine the value of \( P \) which initiates yielding.
b) Determine the maximum value of \( P \).
c) Plot \( P \) vs. the displacement \( \delta \) of the midpoint.
d) If \( P \) is increased up to the maximum value and then decreased to zero, determine the residual stresses in the bars.
C3: Dynamic Systems and Control

PROBLEM C3-1: The new Radisson Diamond uses pontoons and stabilizers to damp out the effect of waves hitting the ship, as shown in Figure 2 (a). The block diagram of the ship’s roll control system is shown in Figure 2 (b).
(a) Obtain the transfer function \( G(s) \) between roll angle and the disturbance, \( D(s) \) (note that reference roll angle is zero, \( R(s) = 0 \)).
(b) Obtain the \( K_2 \) and \( K_3 \) values with which the characteristic roots of \( G(s) \) are located at \( s = -12 \) and \( s = -1 + j \).  
(c) For those \( K_2 \) and \( K_3 \) values found at step (b), draw the Bode gain-phase plots of the transfer function \( G(s) \). Briefly explain how you would read the phase and gain margins from these plots. Point out approximately the frequency band that is undesirable from vibration suppression point-of-view. (Precise numerical values in the Bode plots are not important)
(d) Setting \( K_3 = K_2 = K \), re-write the characteristic equation in the form of \( K \frac{N(s)}{D(s)} = -1 \) and use \( N(s) \) and \( D(s) \) to obtain the root loci of the poles (precise numerical values are not important).

PROBLEM C3-2
(1) Propose an approach to assess the stability of the following characteristic equation with respect to delay \( \tau \). Explain the intermediary steps briefly and state why your approach is feasible. Note that the characteristic equation has a complex coefficient.
(2) \( f(s, \tau) = s + (3 + 3j) e^{-\tau} = 0 \)
(3) When delay \( \tau = 0 \), draw the Nyquist plot using the characteristic function above.
(4) When delay \( \tau = 1 \), draw the Nyquist plot using the characteristic function above.
C4: Finite Element Method

Problem C4-1: The member-12 is fixed at point-1, and rests on the member-23 as shown. And the member-23 is fixed on the ground at point-3. This frame is subjected to the distributed force $q$ shown. ($E = 100$ GPa, $I = 2.5\pi\times10^{-9}$ m$^4$, $A = \pi\times10^{-4}$ m$^2$).

a) Write down (or compute) the relevant element equilibrium equations in the matrix form. State your assumptions clearly.
b) Compute the degrees of freedom at point-2.
c) Find the reaction forces and moments at node-1.

Problem C4-2: Consider the problem of finding the solution to the differential equation:

$$
\frac{d^2}{ds^2} \left[ b(s) \frac{d^2u}{ds^2} \right] + T \frac{d^3u}{ds^3} = f(s) \quad \text{in } 0 \leq s \leq L
$$

a) Using the weak-integral formulation identify the general form of the essential and non-essential BCs of this problem.
b) Using the following specific BCs, obtain the weak integral form of this equation.

at $s = 0 \quad u = \frac{du}{ds} = 0$,
at $s = L \quad b(x) \frac{d^2u}{ds^2} = M_0$ and $\frac{d}{ds} \left[ b(x) \frac{d^2u}{ds^2} \right] = 0$
c) Formulate the element stiffness matrix and the element loading vector for the integral formulation obtained in part b) by,

- Assume that $T = 0$, $b(s) = B = const.$ and $f(s) = F = const.$ and
- Use the following interpolation $u = N_1 u_1 + N_2 \theta_1 + N_3 u_2 + N_4 \theta_2$

$$
N_1 = 1 - 3 \left( \frac{s}{L} \right)^2 + 2 \left( \frac{s}{L} \right)^3, \quad N_2 = s - 2 \frac{s^2}{L} + \frac{s^3}{L^2}, \quad N_3 = 3 \left( \frac{s}{L} \right)^2 - 2 \left( \frac{s}{L} \right)^3, \quad N_4 = -\frac{s^2}{L} + \frac{s^3}{L^2}
$$
Problem C4-3: Show that by collapsing the side 1-2 of the four-node quadrilateral element, a constant strain triangular element is obtained.
Group D: Materials Science and Engineering

D1 Materials Science
- Structure: Bravais lattice, Laue equations, reciprocal lattice, structure factor, order-disorder.
- Crystal Growth: methods, Kelvin equation, thin films, epitaxy.
- Electron energy levels in atoms: quantization, photons, electron configuration.
- Electron energies in solids: band gaps, Brillouin zones, wave vector, metals, insulators, semiconductors, junctions.
- Electrical resistivity: variation with temperature and frequency for metals and semiconductors, mechanisms, superconductivity.
- Thermal resistivity: mechanisms.
- Specific heat: variation with temperature, Debye temperature, phonon spectra.
- Magnetic behavior: types of magnetism, concepts of B, H, M, μ, ψm; B vs H.

References:
- Kasap “Principles of Electronic Materials and Devices”
- Hummel “Electronic Properties of Materials”
- Wilkes “Solid State Theory in Metallurgy”
- Blakemore “Solid State Physics”
- Omar “Elementary Solid State Physics”

D2 Mechanical Behavior of Materials

References:
Hull and Bacon “Introduction to Dislocations” 4th edition, BH

D3 Thermodynamics of Materials

References:
D4 Kinetics of Phase Transformations

References:

D5 Fundamentals of Polymer Science & Engineering
- Structures of Polymers: Bonding, Thermodynamics of Binary Systems; Solubility; Single and Network Molecules.
- Physical States and Transitions: Transition temperatures; Crystallinity, Amorphous Polymers; Phase Separation; Conformations of Single Chains.
- Molecular Weight Distributions/Determinations: Number-Average; Weight-Average; Measurement Distributions.
- Polymers Solutions and Melts: Viscosity; Polymer shapes in Solutions; Effects of concentration, temperature, Normal Stress differences; Rheometry.
- Mechanical Properties: Rubber Elasticity; Viscoelasticity; Effects of molecular weight, crystallinity, and fillers.
- Ultimate Polymer Properties: Creep, Breaking Energy; Fatigue; Stress Relaxation.
- General Properties: Thermal; Density; Hardness.
- Fabrication Processes: Extrusion, Molding, Fibers, Foams.

Sample Problems

D1: Materials Science

\( N_0 = 6.02 \times 10^{23} \text{ mole}^{-1}; \ h = 6.63 \times 10^{-34} \text{ J-s} = 6.63 \times 10^{-27} \text{ erg-sec}; \ c = 3 \times 10^8 \text{ m/s}; \ q = 1.6 \times 10^{-19} \text{ C}; \ k = 1.38 \times 10^{-16} \text{ erg/K} = 1.38 \times 10^{-23} \text{ J/K} = 8.63 \times 10^{-5} \text{ eV/K}; \ R = 1.98 \text{ cal/mole-K}; \ m = 9.11 \times 10^{-31} \text{ kg}. \)

1. Assume that the Debye temperature of a metal is 380 K and the nearest neighbor distance is 0.40 nm
   a. Calculate the Debye frequency.
   b. Calculate the velocity of longitudinal sound waves.
   c. If the sample’s smallest dimension is 10 nm in the direction of the nearest neighbors, sketch the Debye spectrum in this direction.

2. The density of states of free electrons in a metal is given as \( \frac{dN}{dE} \) and is calculated in a reciprocal space. The Debye spectrum for phonons in a solid is given as \( f(\nu) = \frac{dN}{d\nu} \), and is also calculated in a reciprocal space. \( \frac{dN}{dE} \propto E^{1/2} \), and \( f(\nu) \propto \nu^2 \).
   Derive these two density functions to show how they both arise from reciprocal space.

3. a. A superconductor has \( T_c = 70 \text{ K} \) and \( H_c = 0.1 \text{Tesla} \) at 50 K. Calculate \( H_c \) at 40 K.
   b. Explain why a superconductor shows electrical losses at low-frequency alternating current.
   c. For the above superconductor, estimate the electromagnetic wavelength below which the superconductor exhibits normal metal losses.

4. A composite metal alloy consists of equal volume amounts of two phases, \( \forall \) and \( \exists \). \( \forall \) is a solid solution of 30 at.% B in A, and \( \exists \) is a solid solution of 80 at.% B in A. It is known that 10 at.% B in A gives a residual resistance of 1.0 \( \mu \Omega \cdot \text{cm} \); and 90 at.% B in A gives a residual resistance of 2.0 \( \mu \Omega \cdot \text{cm} \).
   Calculate the residual resistance of the two-phased alloy.

5. A germanium sample (\( E_g = 0.7 \text{ eV} \)) at 300 K is lightly doped with phosphorus.
   a. Is this an n – or a p- type material? Please explain.
   b. If the doping concentration is doubled, by how much is the Fermi level changed?
      Is the level increased or decreased?
D2: Mechanical Behavior of Materials

1. (25 Minutes)

Calculate the magnitude of the force acting on a dislocation under pure shear stress $\sigma_{xy} = 100 \text{ MPa}$ whose line vector $S$ is parallel to $(1, 1, 1)$ and the Burgers vector is $(b/2^{1/2}, 0, b/2^{1/2})$. $b = 1 \times 10^{-10} \text{ m}$.

2. (25 Minutes)

(a) What is the Lomer-Cottrell sessile dislocation? Using the Thomson notation, write out the dislocation reaction in FCC crystal between extended dislocations DB on the $\gamma$ plane and BC on the $\delta$ plane give rise to a Cottrell lock. (sketch and identify such dislocation reactions on the Thomson tetrahedron).

(b) What is the Frank Partial dislocation? Using Thomson tetrahedron, sketch and prove the formation of Frank Partial (sessile) dislocations in FCC crystal.

D3: Thermodynamics of Materials

• Show schematically the $G$-X$_B$ curves for all the equilibrium phases in a binary monotectic system at (a) the two invariant reaction temperatures and (b) a temperature at which no liquid exists. (c) Write the names and formulae of the two invariant reactions. (d) How would the monotectic phase diagram change if the element with the higher melting point melted at a much higher temperature?

• If you lived on a planet where temperature is fixed at $727^\circ \text{ C}$, what minimum pressure would you need to do a room-temperature ‘pressure treatment’ to austenitize (transform to fcc) an iron specimen?

Data:  
\begin{align*}
\Delta C_p^{\alpha-\gamma} &= -9.8 - 5.3 \times 10^{-3}T \\
\Delta H^{\alpha-\gamma}(T_0) &= 450 \\
T_0 &= 1183 \\
M_{Fe} &= 55.847 \\
\rho(\gamma-\text{Fe}, 1183K) &= 7.646 \\
\rho(\alpha-\text{Fe}, 1183K) &= 7.538
\end{align*} 

(g/mole)

(g/cm$^3$)

• Starting from the second law, deduce that Gibbs free energy determines the stability of a thermodynamic system under constant P and T. Draw schematically the $G$ vs. $T$ and $G$ vs. $P$ plots of a closed system, e.g., a pure solid metal, and explain why they should be drawn the way you draw them.
• A brazing alloy of composition X₀ (defined in the phase diagram below) is used to braze (join) two plates of an alloy of composition Xₛ (also defined in the phase diagram). The brazing temperature is chosen such that the molten brazing alloy is in thermodynamic equilibrium with the alloy being brazed. Upon cooling, the brazing alloy solidifies under Scheil conditions until temperature falls to the eutectic temperature. At the eutectic temperature the remaining liquid solidifies into a eutectic structure, completing the brazing process. Calculate the volume fraction of the eutectic structure in the solidified brazed joint. What is the average concentration of B in the A-rich solid solution that solidifies prior to the final eutectic solidification?

• Derive appropriate Avrami equations for the overall transformation kinetics for disk shaped precipitates in a thin sheet and cylindrical precipitates in a wire. Assume constant rates of nucleation and growth. Assume also that the disk precipitates in the sheet have their broad faces on the surfaces of the sheet, i.e., they have the same thickness as that of the sheet, and that the cylindrical precipitates in the wire have their axes in the longitudinal direction of the wire and have the same diameter as that of the wire at all time during the transformation.

• Describe and discuss the origin and important features of spinodal decomposition. Use the following terms in your answer and underline them: enthalpy of mixing, curvature of the G curve, uphill diffusion, spontaneous separation, nucleation, activity coefficient, chemical gradient energy, coherency strain energy, miscibility gap, clustering.
1. In polyoxymethylene, the gauche conformations are lower in energy than the trans-conformation.
   a) Provide an explanation of why this is the case for this chain (as fully as you can).
   b) What might be the equilibrium conformation of this chain? Once you know what the equilibrium conformation is, can you predict the shape of the chain, and if so what is the shape of the chain? Hint: Remember that \( G^+ \) or \( G^- \) is the most stable state.
   c) Draw an energy diagram showing the energy as a function of torsional angle for the C-O-bond rotation.

2. Consider a polymer chain with \( N=100 \) beads connected by springs of RMS size \( b=5 \) Å diffusing in a melt with bead friction coefficient of \( \xi=3 \times 10^{-10} \) g/sec at temperature 22 °C.
   a) What is the RMS end-to-end distance \( R \) of this chain.
   b) What is the bet model describing the dynamics of the polymer chain in a melt.
   c) What is the diffusion coefficient \( D \) of the chain.
   d) Estimate the longest relaxation time (\( \tau \)) of the chain.
   e) Estimate the viscosity of the melt.

3. Estimate the configurational entropy changes that occur under the following circumstances:
   a) 500 g of toluene are mixed with 500 g of styrene monomer.
   b) 500 g of toluene are mixed with 500 g of polystyrene, \( M_w=100,000 \) g/mol.
   c) 500 g of polystyrene, \( M_w=100,000 \) g/mol, are mixed with 500 g of poly(phenylene oxide), \( M_w=100,000 \) g/mol. (This is a rare example where two high molecular weight polymers are soluble in each other).
   d) What does your result for (c) indicate about the thermodynamic requirement for polymer-polymer solubility?

4. Assign a polymer from polycarbonate (PC), PMMA, nylon-6,6 and polyoxymethylene (POM) to each of the following set of characteristics: (a) High melting point, high strength, good frictional properties and resistance to fatigue, (b) Highly amorphous, rigid and transparent, but poor impact strength, (c) Strong and tough, but prone to water absorption, and (d) Highly amorphous, rigid, transparent, and high impact strength.

5. Draw a typical molecular weight distribution curve and label each of the following molecular weights: (i) number-average molecular weight (ii) weight-average molecular weight (iii) viscosity-average molecular weight.
   a) Define mathematically the number-average and weight-average molecular weights. When does the number-average molecular weight equal the weight-average molecular weight?
   b) Define the first and second order phase transitions in polymers and give one experimental method to differentiate between them.
Group E
E1 Design and CAD/CAM
   Stress and Strain
   Deflection and Stiffness
   Steady and Variable Loading
   Design of Mechanical Elements; e.g.
      - Springs
      - Gears
      - etc.
   Wireframe Modeling
   Surface Modeling
   Solid Modeling
   Geometric Transformations

   Mechanical Assemblies
   Mechanical Tolerancing
   Mass Property Calculations
   Part Programming

1. (20 minutes)

A parallel helical gearset consists of a 19-tooth pinion driving a 57-tooth gear. The pinion has a left-hand helix angle of 20°, a normal pressure angle of 14 1/2°, and a normal diametral pitch of 10 teeth/in. Find:

a) The normal, transverse, and axial circular pitches
b) The transverse diametral pitch and the transverse pressure angle
c) the addendum, dedendum, and pitch diameter of each gear

Note:

\[ P_n = P_t \cos \phi \]

\[ P_x = \frac{P_t}{\tan \omega} \]

\[ P_n = \frac{P_t}{\cos \omega} \]

\[ \cos \phi = \frac{\tan \phi_n}{\tan \phi}. \]

\[ a = \frac{1}{P_n}, \quad b = \frac{1.25}{P_n} \]

2. (20 minutes)

For the three points shown, find:

a) The equation of the closed Hermite cubic spline that passes through point \( P_0 \) and tangent to the line segments \( P_0 P_1 \) and \( P_2 P_0 \).

b) The equation of the closed Bezier curve that passes through point \( P_0 \).

c) Are the two curves identical?

Note:

\[ P(u) = (2u^3 - 3u^2 + 1) P_0 + (-2u^3 + 3u^2) P_1 + (u^3-u^2+u) P_0 + (u^3-u^2) P_1 \]

\[ \circ \leq u \leq 1 \]

\[ P(u) = \sum_{i=0}^{n} P_i \left[ \frac{n!}{i!(n-i)!} \right] u^i (1-u)^{n-i} \]

\[ \circ \leq u \leq 1 \]
Group F

F1 Human-Machine Systems


Preparation:
- Laboratory experience in human factors.


F1 Human Machine Systems

1. (90 minutes)

The human-computer interface has become one of the most prevalent forms of human-machine interfaces.

a. Describe the history and development of the human-computer interface emphasizing those facets that impact the effectiveness of human performance.

b. What are some changes in the human-computer interface that may happen in the next five years?

c. Consider the Human Factors issues and concerns associated with the new interfaces in your answer to part b.

d. Describe how you would resolve the issues/concerns listed in your answer to part c.

2. (20 minutes)

Research within the field of Human Factors Engineering creates some special problems for the human factor engineer. Discuss the considerations that arise in relation to the use of human subjects in human factors research.
Group G

G1 Manufacturing Systems
- Manufacturing Methods and Processes
- Structures of Material (metals and polymers)
- Systems modeling
- Simulation of manufacturing systems
- Design and development of facilities
- Group technology
- Part coding and classification
- Numerical control
- Computer aided manufacturing
- Computer aided design.


G2 Production and Logistics
- Forecasting
- Aggregate planning
- Scheduling
- Inventory analysis and control
- Manufacturing resource planning
- Production life cycle planning
- Project management
- Systems reliability and testing
- Systems maintenance
- Needs analysis and requirements definition
- Logistics in systems design
- Economic evaluation of systems

Project Management by Meredith and Mantel, John Wiley; Project Management by Kerzner, Van Nostraml Reinhold; Logistics Engineering and Management by Blanchard, Prentice Hall; Facilities Planning by Tompkins and White, et al, John Wiley; Engineering Economy by Thuesen and Fabrycky, Prentice Hall; and IDEF Standards Documents, MIME 3217 class notes, Gnomon Copy
1. (20 minutes)

Develop the form code (first five digits) in the Optiz system for the part illustrated in the following figure. The table of form code in the Optiz system is attached for your reference.

![Diagram of a part with dimensions](image)

2. (30 minutes)

The shear strength of a certain work material = 50,000 lb/in². An orthogonal cutting operation is performed using a tool with a rake angle = 20° at the following cutting conditions:

- speed 100 ft/min,
- chip thickness before the cut = 0.015 in.,
- width of cut = 0.150 in.

The resulting chip thickness ratio = 0.50. Determine the following parameters of the orthogonal cutting operation:

- a. Shear plane angle
- b. Shear force
- c. Cutting force
- e. Thrust force
- f. Friction force
<table>
<thead>
<tr>
<th>Digit 1</th>
<th>Digit 2</th>
<th>Digit 3</th>
<th>Digit 4</th>
<th>Digit 5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Part class</strong></td>
<td><strong>External shape, internal shape elements</strong></td>
<td><strong>Internal shape, internal shape elements</strong></td>
<td><strong>Plane surface machining</strong></td>
<td><strong>Auxiliary holes and gear teeth</strong></td>
</tr>
<tr>
<td>0</td>
<td>Smooth, no shape elements</td>
<td>No hole, no breakthrough</td>
<td>No surface machining</td>
<td>No auxiliary hole</td>
</tr>
<tr>
<td>1</td>
<td>0.5 &lt; L/D &lt; 3</td>
<td>No shape elements</td>
<td>Surface plane and/or curved in one direction, external</td>
<td>Axial, not on pitch circle diameter</td>
</tr>
<tr>
<td>2</td>
<td>Thread</td>
<td>No shape elements</td>
<td>External plane surface related by graduation around a circle</td>
<td>Axial on pitch circle diameter</td>
</tr>
<tr>
<td>3</td>
<td>Functional groove</td>
<td>No shape elements</td>
<td>External groove and/or slot</td>
<td>Radial, not on pitch circle diameter</td>
</tr>
<tr>
<td>4</td>
<td>No shape elements</td>
<td>Smooth or stepped to one end</td>
<td>External spline (polygon)</td>
<td>Axial and/or radial and/or other direction</td>
</tr>
<tr>
<td>5</td>
<td>Thread</td>
<td>Smooth or stepped to one end</td>
<td>External plane surface and/or slot, external spline</td>
<td>Axial and/or radial on PCD and/or other directions</td>
</tr>
<tr>
<td>6</td>
<td>Functional groove</td>
<td>Functional groove</td>
<td>Internal plane surface and/or slot</td>
<td>Spur gear teeth</td>
</tr>
<tr>
<td>7</td>
<td>Functional cone</td>
<td>Functional cone</td>
<td>Internal spline (polygon)</td>
<td>Bevel gear teeth</td>
</tr>
<tr>
<td>8</td>
<td>Operating thread</td>
<td>Operating thread</td>
<td>Internal and external polygon, groove and/or slot</td>
<td>Other gear teeth</td>
</tr>
<tr>
<td>9</td>
<td>All others</td>
<td>All others</td>
<td>All others</td>
<td>All others</td>
</tr>
</tbody>
</table>

Table 1. Form code (digits 1 through 5) for rotational parts in the Opitz system. Part classes 0, 1, and 2.
G2 Production and Logistics

1. (45 minutes)

Construct a project network with the following precedence relationships:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Immediate Successor</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Optimistic</td>
</tr>
<tr>
<td>A</td>
<td>E, F</td>
<td>8</td>
</tr>
<tr>
<td>B</td>
<td>C, D, E, F</td>
<td>0.5</td>
</tr>
<tr>
<td>C</td>
<td>H</td>
<td>16</td>
</tr>
<tr>
<td>D</td>
<td>G, H</td>
<td>3</td>
</tr>
<tr>
<td>E</td>
<td>G, H</td>
<td>7</td>
</tr>
<tr>
<td>F</td>
<td>-</td>
<td>0.5</td>
</tr>
<tr>
<td>G</td>
<td>-</td>
<td>8</td>
</tr>
<tr>
<td>H</td>
<td>-</td>
<td>0.75</td>
</tr>
</tbody>
</table>

(a) Find the critical path and project duration of this project.
(b) Determine the probability of completing the project 3 days earlier.
(c) Determine the sequence of activities that constitute the second and third critical paths, that is, the paths of the second and third longest duration.
(d) Based on the first, second and third critical paths determine the four most critical activities. Explain your reasons for choosing these four activities.
(e) Find a time, say \( x \), such that the probability of project completion time exceeding \( x \) is less than approximately 0.10.
(f) If the available resources for this network are 3 units and the resources required for activities A through H are:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Resources Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
</tr>
<tr>
<td>F</td>
<td>2</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
</tr>
<tr>
<td>H</td>
<td>1</td>
</tr>
</tbody>
</table>
G2 Production and Logistics Cont.d

i. Find the duration to complete all activities using the ACTIM approach.
ii. Find the duration to complete all activities using TIMRES approach.

2. (15 minutes)

An item is manufactured in batches within a manufacturing facility and the following data is applicable:

- Sumption rate: \( \text{items/month} \)
- Fluctuation rate: \( \text{items/month} \)
- Age costs (based on average inventory): \( \text{per unit-year} \)
- Rest charges: \( \text{per unit-year} \)
- Setup charges per batch: 

The batch is produced so that it is completed exactly when the previous batch is depleted. Determine the EMQ value and associated inventory costs per unit.
Group H

**H1 Operations Research**
- Linear Programming
- Networks
- Dynamic Programming
- Markovian Models
- Basic Queuing Models
- Basic Inventory Models


**H2 Reliability and Quality Assurance**
- Quality planning, control and improvement.
- Process control, discrete and continuous control charts.
- Moving average and custom control charts.
- Modified control charts.
- Discrete and variable sampling methods, Mil standards.
- Process capability analysis.
- Quality engineering method of robust design.

Mathematical definitions of reliability, hazard rate, intensity function, failure rate and availability.
Stress and strength analysis, reliability block design, fault tree method.
Network reliability methods, Markovian methods, reliability testing.
Reliability estimates from field and test data.
Confidence interval on reliability.
Maintenance and replacement policies.


**H3 Simulation**
- Simulation modeling
- Simulation languages
- SIMAN simulation language
- Input data analysis
- Model verification and validation
- Output analysis
- Model experimentation and optimization
- Random number generation
- Random variate generation

1. (50 minutes)

Consider the following linear programming problem:

Minimize \( z = x_1 - x_2 \)
subject to
\[
\begin{align*}
3x_1 + 2x_2 &\leq 8 \\
x_1 + 2x_2 &\geq 5 \\
3x_1 + x_2 &\geq 0 \\
x_1 &\text{ is unrestricted in sign} \\
x_2 &\geq 0
\end{align*}
\]

a. Construct the feasible region on a graph. Using the graph,
   (i) Determine the optimal point(s) and the optimal objective function value,
   (ii) Answer the previous question (I) If the coefficient of \( x_1 \) in the objective function is
   \((1/2)\) instead of 1 and all other coefficients remain unchanged.

b. Consider the solution \((x_1=1, x_2=2)\).
   (i) Is this a feasible solution? Justify your answer,
   (ii) If yes, does it correspond to a basic solution of the problem? Justify your answer.

c. Construct the Phase I initial Simplex tableau and perform a single iteration.

d. Write the dual of the problem.

2. (40 minutes)

A telephone exchange has seven lines. Calls arrive at a mean rate of two per minute and the interarrival time is exponentially distributed. Conversations have also an exponential distribution with mean four minutes. When all seven lines are occupied the caller receives a busy signal (call doesn’t go through and the caller hangs up).

a. Identify an appropriate model for this queueing system, determine its parameters and draw its rate diagram.

b. What is the probability that
   (i) a conversation will take more than five minutes?
   (ii) at least two calls arrive in the system within a minute?

For the remaining questions of this problem no numerical answers are required; however, you should indicate formulas to be used and values to be assigned to symbols in order to arrive at the solution. You don’t have to do any calculations. If a formula requires the use of a symbol already used, just refer to it without writing its formula again.

At steady state,
HI Operations Research Cont.d

c. What proportion of time
   (i) three or less lines are free?
   (ii) an incoming call receives a busy signal?
d. What is the expected number of idle lines?
e. What is the utilization of the telephone exchange?
f. What is the average number of calls per hour
   (i) processed by the telephone exchange?
   (ii) lost through busy signal?
g. The telephone exchange is going to be redesigned. Show how you can determine the number of lines required so that at least 95% of incoming calls are captured by the telephone exchange?

H2 Quality Assurance & Reliability Engineering

1. (30 minutes)

Specifications on a bearing diameter are established at 8.0 ± 0.01 cm. A sample of size n = 8 is used, and a control chart shows statistical control, with the best current estimate of the population standard deviation of 0.001.

a. Use a process capability index that would be most suitable for capability analysis and describe why you chose this index.
b. If the fraction of non-conforming product that is barely acceptable is 0.135, find the 3-sigma limits on the modified control chart for this process.
c. What conclusions can be drawn from your results (a and b).

2. (45 minutes)

Suppose the failure density function for a class of components (system) is:

\[ f(t) = 0.25 - (0.25/8)t, \quad 0 < t < 8 \]

where \( t \) is in years.

a. Find hazard function.
b. Use your result of part (a) to find the reliability function.
c. Sketch the three functions in one figure and comment about the performance of this system over time.
d. Find the MTTF.
e. Find the median time to failure for the system.
f. Compare the comment on your results of part d and f. Which is the better representation of system performance? Why? Quantify our comment.
H3 Simulation

1:
(a) (50 minutes)
A certain small island is connected to the mainland by a single ferry that shuttles back and forth between the hours of 6:00 AM and 1:00 AM. Lately, there have been complaints about long waits at the ferry landings. To explore the possible need for a faster ferry or different operating procedures, simulation analysis is to be performed. A detailed description of the ferry operation follows.

Vehicles arrive at both mainland and island ferry docks every 4.5 minutes, on the average, according to an exponential distribution. There may be some variation in mean arrival rate over the hours of operation, but initially it may be ignored. When the ferry arrives, vehicles board the vessel one at a time until the ferry is full (capacity is 35 vehicles) or there are no more vehicles waiting to board. The crossing takes an average of 15 minutes distributed normally with a standard deviation of 3 minutes. When the ferry arrives at its destination, vehicles disembark one at a time. Boarding by waiting vehicles can begin only after all cars have disembarked.

Write the SIMAN V model and experiment code to represent the ferry operation. Include in the output, measures of the waiting time and queue size at the island and mainland ferry landings. Your code should reflect a model run that represents one week's operation. One rather important issue to be concerned about when constructing your model: be certain that the ferry is actually at the dock before you permit vehicles to board or disembark!

(b) (20 minutes)
A renowned simulation expert has challenged the conclusions you have drawn using your model output. He mumbled something about terminating vs. nonterminating analysis and steady-state vs. transient output. Are these things of relevance to your use of the model? Relate them specifically to the ferry operation.

(c) (20 minutes)
Once the issues of part (b) have been resolved, the question of how to use your model to develop recommendations for the ferry company remains. Define some alternative courses of action to be explored using your model and specify a well-structured, explicit experiment to be used for their exploration.

Note: For this question, a SIMAN Language primer will be provided to students at the time of the examination.
H3 Simulation Cont.d

2:
(a) (50 minutes)

Consider the intersection of two two-lane roads, one running north-south road and one running east-west. Cars going east-west arrive at the intersection according to an exponential distribution with a mean of 10 seconds, while north-south arrivals are exponential with a mean of 28 seconds, since this is a secondary road. A traffic light at the intersection controls traffic by cycling through red and green signals at a constant rate. Given the dimensions of the intersection, only two cars may be present in it at the same time. All cars take four seconds to cross the intersection. Cars heading east-west turn right or left with a probability of 0.2, while cars traveling north-south turn right or left with a probability of 0.5. Cars require 6 seconds to negotiate turns. It desired to keep the queue of cars traveling east-west at the traffic signal to a maximum of 8 cars in either direction.

Write appropriate SIMAN V code to model this intersection and measure queue lengths.

(b) (20 minutes)

What should the time between red-green signal cycles be to satisfy the maximum queue lengths specified in Part (a)? Design an appropriate experiment to use the model to answer this question.

(c) (time: 20 min.)

Pseudo random numbers with desirable properties are vital to successful simulation analysis. One such property is the period of the random number stream. Given the following pseudo random number generating algorithm, what is its maximum possible period? Will this be achieved? Why or why not?

\[ Z_i = (aZ_{i-1} + c)(\text{mod } m) \]

where:
\[ a = 6 \]
\[ c = 2 \]
\[ m = 16 \]
Group I

11 Software Engineering
The software life cycle (requirements analysis specification, software design, coding, testing, maintenance).
Verification, validation and documentation at various stages of the life cycle.
Overview of user interface design, prototyping, CASE tools, software metrics, and software development environments.
Structured analysis and structured design.
Object-oriented analysis and object-oriented design.
Real-time software design.
Software testing and software quality assurance.
Software project management.


12 Programming Languages
Comparative aspects of programming languages such as control structures, parameter passing conventions, run-time structures and binding.
Historical survey of programming languages.
Coverage of traditional, object-oriented, logical and function languages such as Ada, C++, Prolog, LISP, and Java.


13 Artificial Intelligence in Engineering
Knowledge representation (semantic networks, frames, production rules, logic systems).
Problem solving methods (heuristic search algorithms, forward and backward chaining, constraint handling, truth maintenance).
Approximate reasoning methods (Bayesian, Dempster-Shafer, fuzzy logic, certainty factors).

I1 Software Engineering

1. (25 minutes)

Describe in your own words at least three of the known software engineering paradigms (software process models). Support your prose with figures. Discuss advantages and disadvantages of each paradigm.

2. (20 minutes)

What kind of metrics would you use to measure the software project? What metrics would you use to measure the process? Describe examples of both kinds of metrics.

I2 Programming Languages

1. (90 minutes)

Ada is considered to be a modern imperative language.

a. Describe the history of Ada, including why it was developed.
b. Ada supports concurrency. How does it do this? Give an example.
c. How does Ada support object-oriented concepts?
d. What are the differences between Ada and C++ in terms of object-oriented programming?

2. (90 minutes)

You have been awarded (by AT&T) a million dollar contract to develop a new programming language. This new language does not have to be backwards compatible with any existing programming language. I am sure you are aware that the very successful C programming language was developed at Bell Labs. It is hoped that this new language will also become very popular.

a. Describe the features you would put into your new language.
b. Discuss the details of how these features would work, using pseudo code where appropriate.
c. Present both sides of “trade offs” you made when selecting your language’s features.
d. Rank the features in order of importance and justify your ranking.
d. What are the overall goals of your new programming language?
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1. (20 minutes)

Represent the following sentences in first-order logic, using a consistent vocabulary which you must define.

a. To achieve fuel economy, light materials should be selected for constructing the parts.
b. Not all light materials have good strength.
c. Plastic materials are lighter in weight but lack strength.
d. When strength and light weight are desired, composite materials should be used.
e. Composite materials are very difficult to process.

2. (60 minutes)

What is given in the following figure is a traveling salesman problem. Assume that the salesman must start from the Airport, cover all other three locations, and return to the airport. The salesman can visit each location only once. Find out the heuristically optimal travel sequence using Hill-Climbing and Best-First search algorithms. Find out also the optimal travel sequence using Depth-First algorithm. Compare the results given by the three algorithms.